Lambek-Grishin Calculus: Focusing, Display and Full Polarization Giuseppe Greco¹, Valentin D. Richard², Michael Moortgat¹

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Legends: State of the art •Contribution

1. Basic Lambek-Grishin calculus (LG) [Moo09, MM12]

Residuated non-associative non-commutative tensor

 $B \leq A \setminus C$ iff $A \otimes B \leq C$ iff $A \leq C / B$

+ Co-residuated non-associative non-commutative cotensor

 $C \oslash B \leq A$ iff $C \leq A \oplus B$ iff $A \otimes C \leq B$

2. Weakening relation [KMJ19] $\leq : \mathcal{A} \rightarrow \mathcal{B}$ Relation $\preccurlyeq \subseteq \mathcal{A} \times \mathcal{B}$ on two ordered sets such that

$A' \leq_{\mathcal{A}} A A \leq B B \leq_{\mathcal{B}} B'$ $A' \leq B'$

Interprets heterogeneous sequents

Orders are homogeneous weakening relations

•3. Fully polarized LG algebra (FP.LG)

- 4 ordered sets and 5 heterogeneous weakening relations
- Shifts \downarrow , \uparrow and their adjoints \uparrow , \downarrow



• LG operations (§1.) and their ℓ , *r*-variants (§4.)

[And01]	Jean-Marc Andreoli. Focussing and proof construction. Annals of Pure and Applied Logic, 107(1):131 – 163, 2001.		Algebraic proof theory for LE-logics, submitt
		[KMJ19]	Alexander Kurz, Andrew M. Moshier, and Ac Stone duality for relations, 2019.
[Bas13]	Arno Bastenhof. Focalization and phase models for classical extensions of non-associative Lambek calculus, 2013.	[Lau17]	Olivier Laurent. A proof of the focusing property of linear log Unpublished note, 2004, revised 2017.
[FGK+16]	Sabine Frittella, Giuseppe Greco, Alexander Kurz, Alessandra Palmigiano, and Vlasta Sikimić. Multi-type sequent calculi. In A. Indrzejczak, J. Kaczmarek, and M. Zawidski, editors, <i>Proceedings of Trends in Logics XIII</i> , pages 81–93. Łódź	[MM12]	Michael Moortgat and Richard Moot. Proof nets and the categorial flow of informa In A. Baltag, D. Grossi, A. Marcoci, B. Roder S. Smets, editors, <i>Logic and Interactive RAte</i> 2011, pages 270–302. ILLC, University of A
[GJL+19]	University Press, 2016. Giuseppe Greco, Peter Jipsen, Fei Liang, Alessandra Palmigiano, and Apostolos Tzimoulis.	[Moo09]	Michael Moortgat. Symmetric categorial grammar. Journal of Philosophical Logic, 38(6):681–7 ⁻

(1)

(2)

(3)

 All cut-free* derivations of focused **Display** Lambek Grishin calculus (fD.LG) are strongly focalized

and

•fD.LG is sound and complete w.r.t. fully polarized Lambek-Grishin algebras with heterogeneous consequence relations.

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710, 2009.



- LG-connectives

Strongly focal-





Figure: Diagram of phase transitions. Represented: shift rules (\xrightarrow{n}) , invertible rules (\leftrightarrow), tonicity rules (teleporters $-\infty$) $, \rightarrow$), non-focused sequents in the grey rectangle, focused sequents outside of it

everyone ⊢ **s** /́↓sleeps Ŝч everyone ⊗↓sleeps ⊢ **s** everyone⊗↓sleeps ⊫́↓́**s** everyone⊗↓sleeps ⊫↓**s**↓R Figure: Derivation of Everyone sleeps in fD.LG.