Dynamic Effects of Modalized Questions

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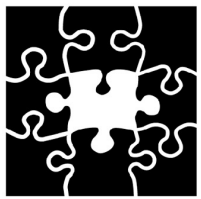
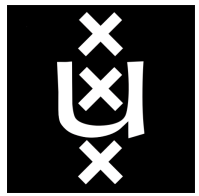
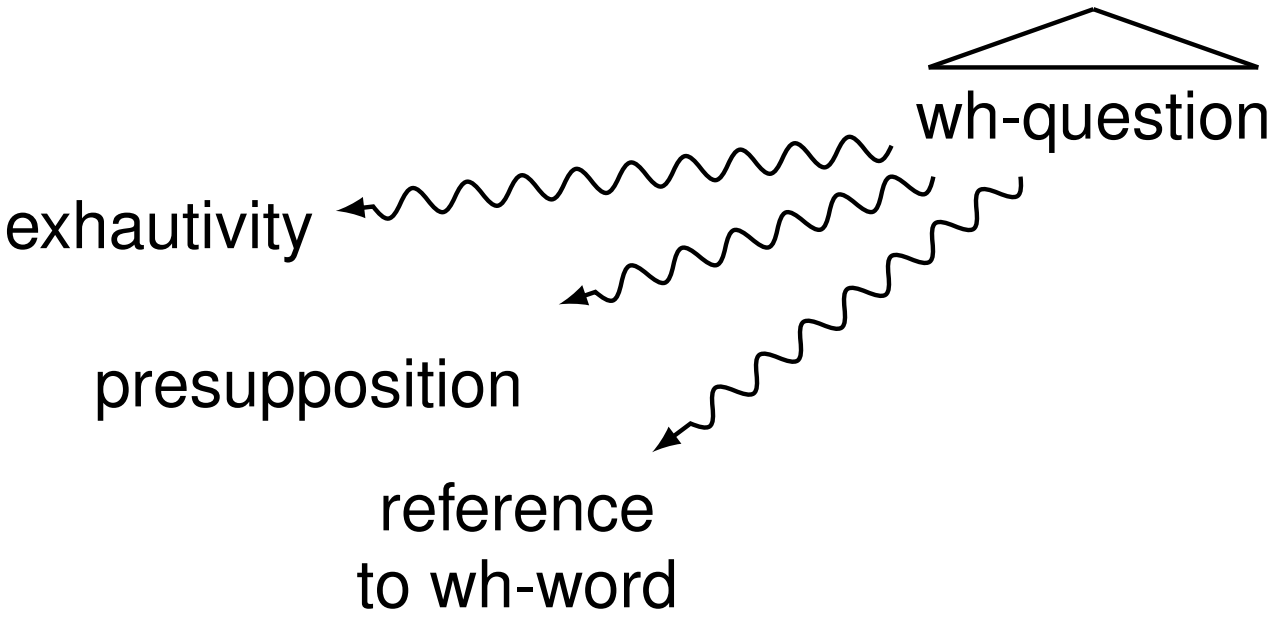




Topic of this talk



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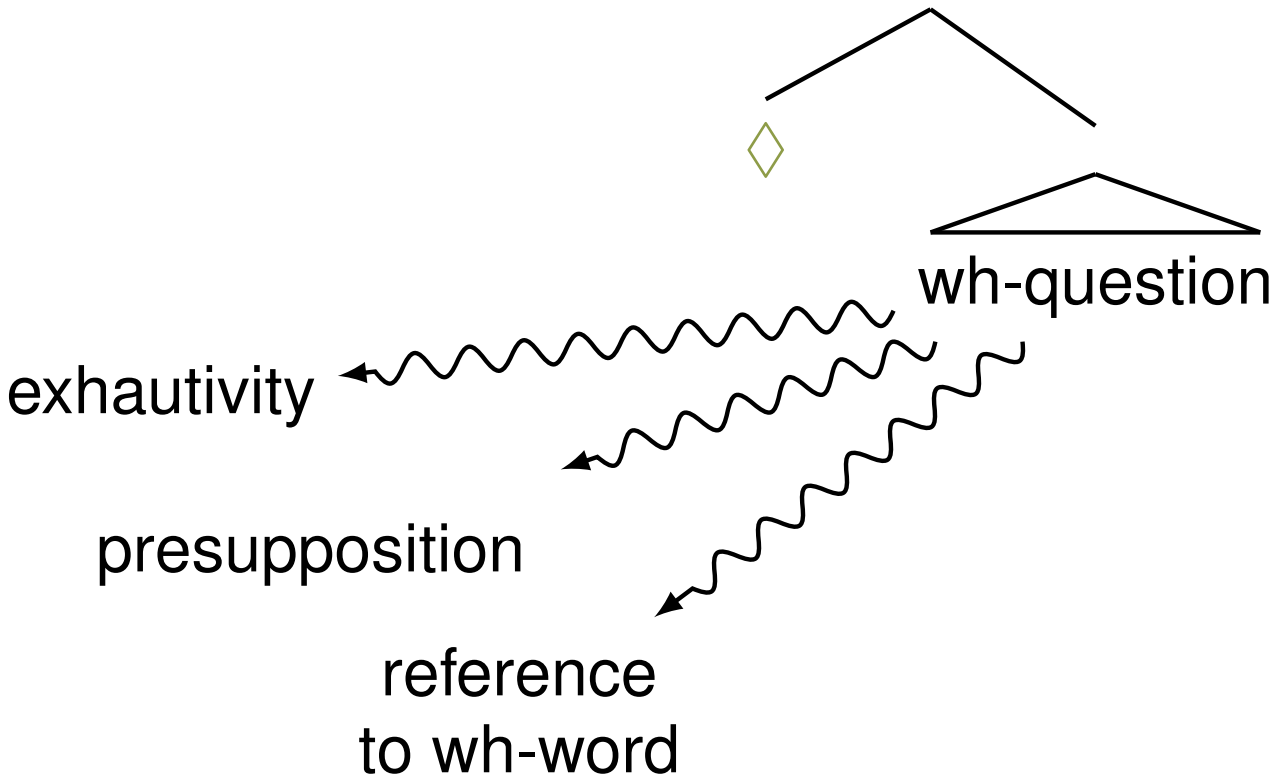

Modalized question :

(1) Which student might come tonight ?

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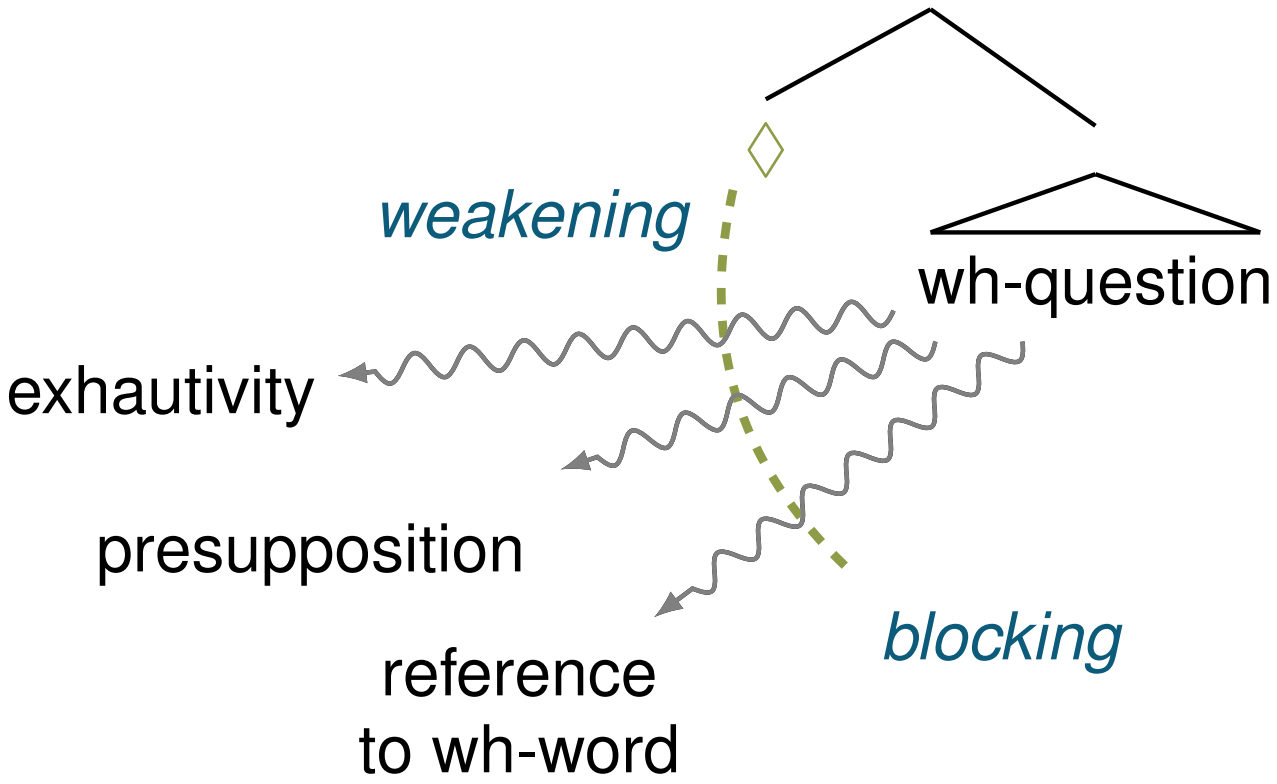

Modalized question :

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Exhaustivity and presupposition weakening

(2) SITUATION : I am hiding some part of English word FO 🤚 M
It has 4 or 5 letters.

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(3) a. Which letter is hidden here ?
$\overset{\text{presup}}{\Longrightarrow}$ *There exists exactly one (token) letter which is hidden.*
b. Either “A” (for FOAM)
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- Strongly exhaustive
- Global uniqueness presupposition

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(4) a. Which letter could be hidden here ? (HIRSCH et SCHWARZ 2019)
$\overset{\text{presup}}{\Longrightarrow}$ *(For all worlds,) It might be that exactly one letter is hidden.*
b. “A”
c. “R”
d. “A and/or R”

- Question about the content of the English lexicon
- **Mention-some**
- **Local uniqueness presupposition**

Modal subordination

(5) a. A : Which^{u} university might want to host the next ACL conference ?
(*might* $>$ *which* reading)

b. B : ?I don't know, but it$_{u}$ needs a lot of support.

Modal subordination

(5) a. A : Which^u university might want to host the next ACL conference ?
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b. B : ?I don't know, but it$_u$ needs a lot of support.

c. B : I don't know, but it$_u$ would need a lot of support.

- $\Diamond$ is **externally static**
- but allows **modal subordination**

Goal

Define ◊ in Dynamic Inquisitive Semantics :

- weakening exhaustivity
- weakening uniqueness presupposition
- externally static
- allowing modal subordination

Kripkean existential modality

- Accessibility relation R

Intensional :

$$[\![\Diamond\varphi]\!]_{\text{int}} = \{w \in W \mid \exists w'.\ w\ R\ w' \wedge w' \in [\![\varphi]\!]_{\text{Int}}\} = \text{R}^{-1}([\![\varphi]\!]) \quad (1)$$

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CIARDELLI 2016's inquisitive existential modality :

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- InqBK problem : $\Diamond$ is an inquisitive plug
- Solution : inverting the quantifiers
- s resolves $\Diamond\varphi$ iff s resolves some $\Diamond\psi$, where ψ is a proposition resolving φ.

Example

$$[\![\Diamond\varphi]\!]_{\mathrm{MInqB}} = \{s \subseteq W \mid \exists s' \in [\![\varphi]\!]_{\mathrm{MInqB}}.\ s \subseteq \mathrm{R}^{-1}(s')\} \tag{5}$$

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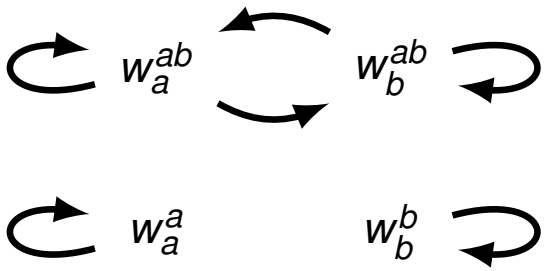
- Arrows represent the accessibility relation R : Mary's epistemic state



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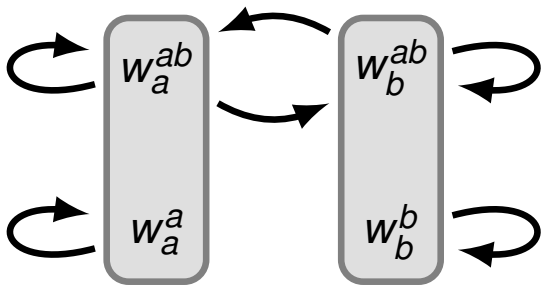

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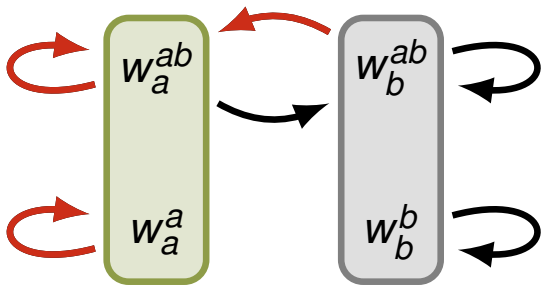

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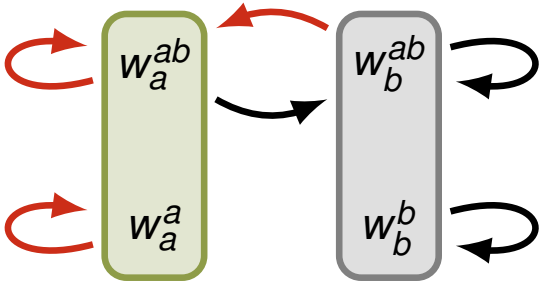

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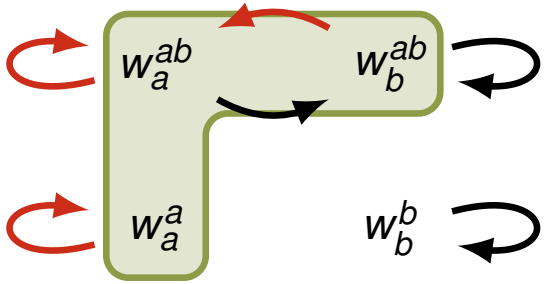

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- R^{-1} is widening the alternatives (*if* R *is an equivalence relation*)

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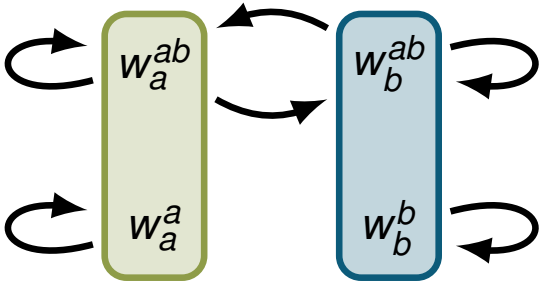

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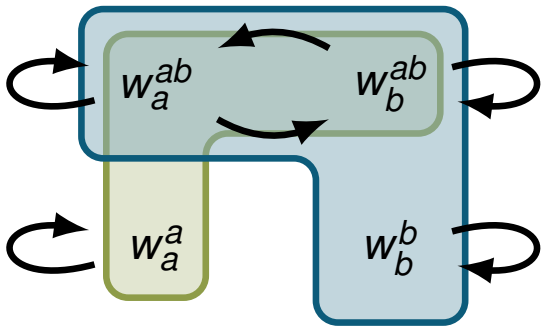

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Extension to dynamic inquisitive semantics

In GROENENDIJK, STOKHOF et VELTMAN 1996 (GSV),

- information state s : set of pairs $\langle w, g \rangle$ of world and assignment
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$$[\![\Diamond \varphi]\!]_{\mathrm{MGSV}} = s \mapsto \{\langle w, g \rangle \in s \mid w \in \mathrm{R}^{-1}(\mathrm{WC}([\![\varphi]\!]_{\mathrm{MGSV}}(s)))\} \tag{6}$$

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Combining it all : Dynamic Inquisitive Semantics (DOTLAČIL et ROELOFSEN 2021)

$$\Diamond \mathcal{U} = c \mapsto \{\boldsymbol{s \in c} \mid \exists \boldsymbol{s' \in \mathcal{U}(c).\ \mathrm{WC}(s) \subseteq \mathrm{R}^{-1}(\mathrm{WC}(s'))}\} \tag{7}$$

- Externally static
- Similar to $[\![\Diamond \varphi]\!]_{\mathrm{MInqB}}$

Pushing and percolating : intuitions

Goal update of common ground :

$$\textit{might}\ \varphi, \textit{would}\ \psi \quad \rightsquigarrow \quad \Diamond\varphi \wedge \Box(\varphi \rightarrow \psi) \tag{8}$$

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$\langle c \rangle$

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$$\langle c \rangle \xrightarrow{\text{PUSH } \varphi} \left\langle \begin{array}{c} c[\varphi] \\ c \end{array} \right\rangle \xrightarrow{\text{PERC } \Diamond\varphi} \left\langle \begin{array}{c} c[\varphi] \\ c[\Diamond\varphi] \end{array} \right\rangle$$

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might φ / could φ

$$\xrightarrow{\text{PUSH } \psi} \left\langle \begin{array}{c} c[\varphi][\psi] \\ c[\varphi] \\ c[\Diamond\varphi] \end{array} \right\rangle \xrightarrow{\text{PERC } \psi} \left\langle \begin{array}{c} c[\varphi][\psi] \\ c[\varphi][\psi] \\ c[\Diamond\varphi][\Box(c[\varphi] \to \psi)] \end{array} \right\rangle$$

would ψ

Example : setting

(7) a. Which^{u} letter could be hidden in FO_M ?

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- Functional heads Type (here : Int) and Focus
- *could* rises to spec FocusP (Mari & Giannakidou p.c.)

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World w	w_A^A	w_A^*	w_{RU}^*	w_R^*	w_R^R	w_{RU}^{RU}
Words with skeleton FO_M in English	FOAM	FOAM, FORM, FORUM	FOAM, FORM, FORUM	FOAM, FORM, FORUM	FORM	FORUM
Actual word on the board	FOAM	FOAM	FORUM	FORM	FORM	FORUM
Accessible worlds	$\{w_A^A\}$	$\{w_A^*, w_R^*, w_{RU}^*\}$	$\{w_A^*, w_R^*, w_{RU}^*\}$	$\{w_A^*, w_R^*, w_{RU}^*\}$	$\{w_R^R\}$	$\{w_{RU}^{RU}\}$

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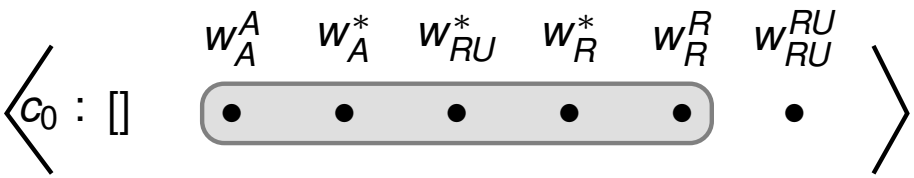

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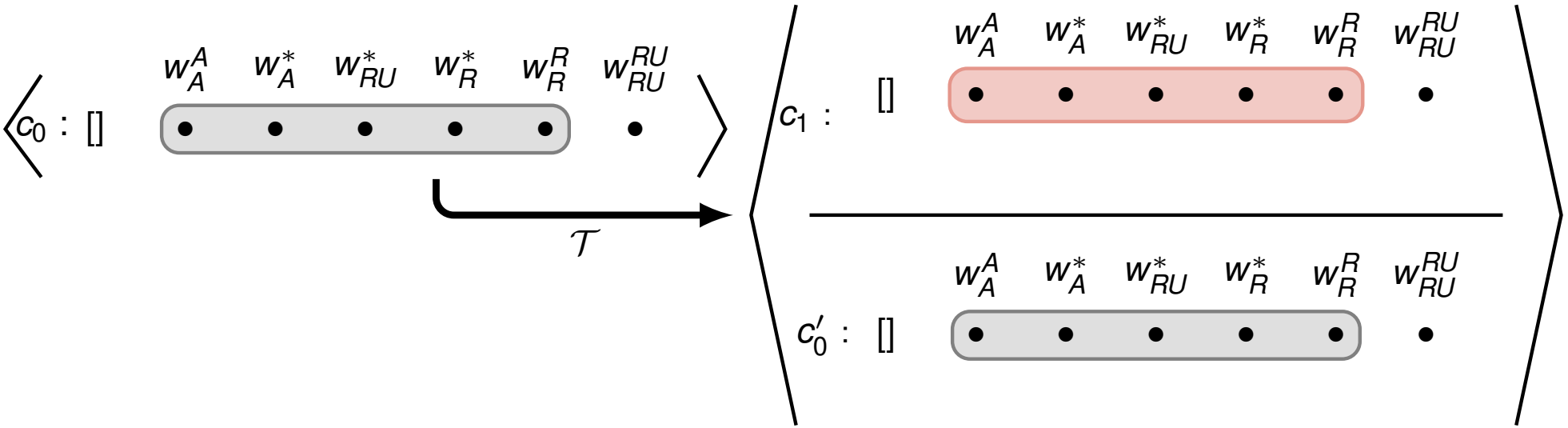

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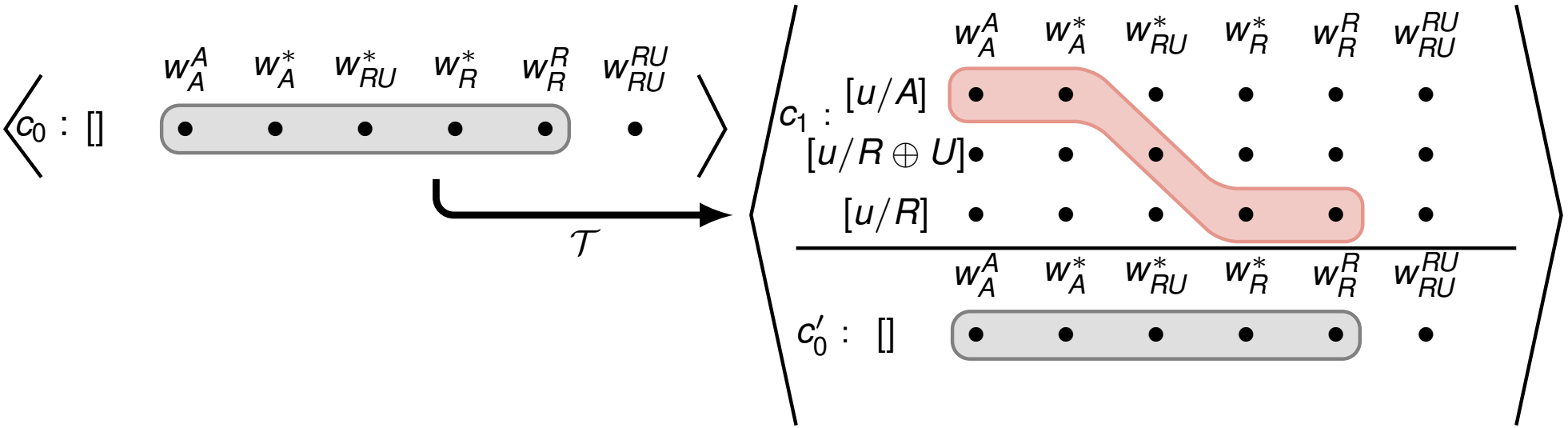

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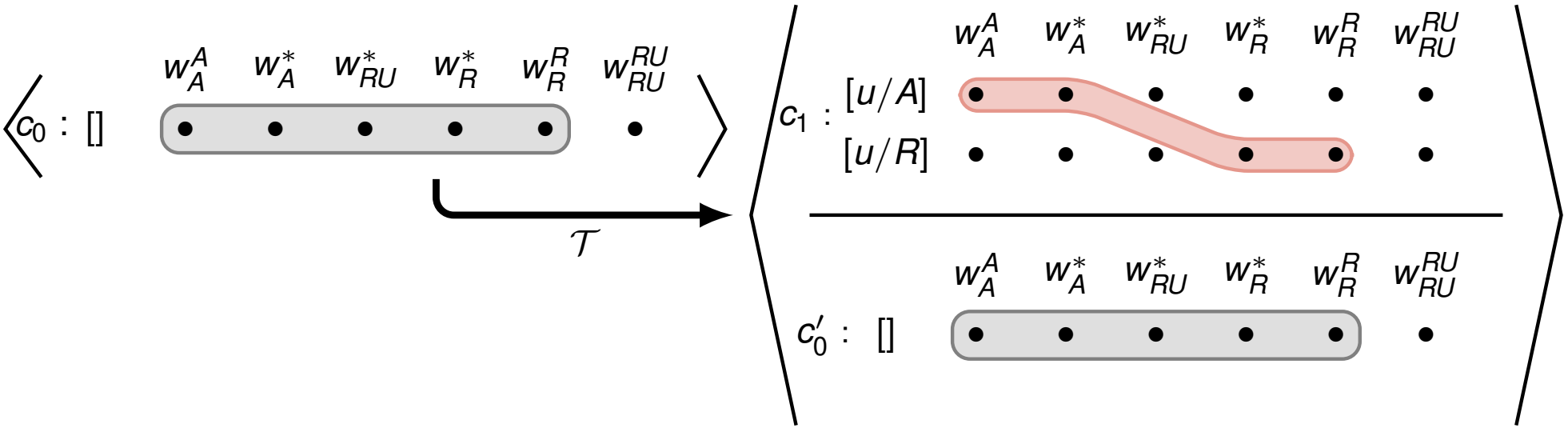

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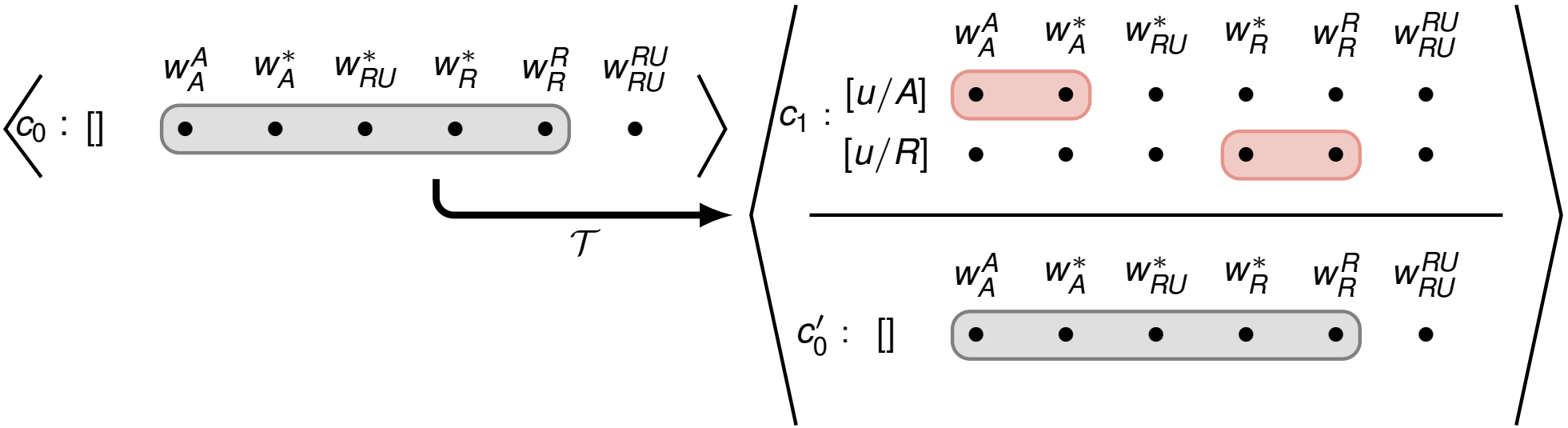

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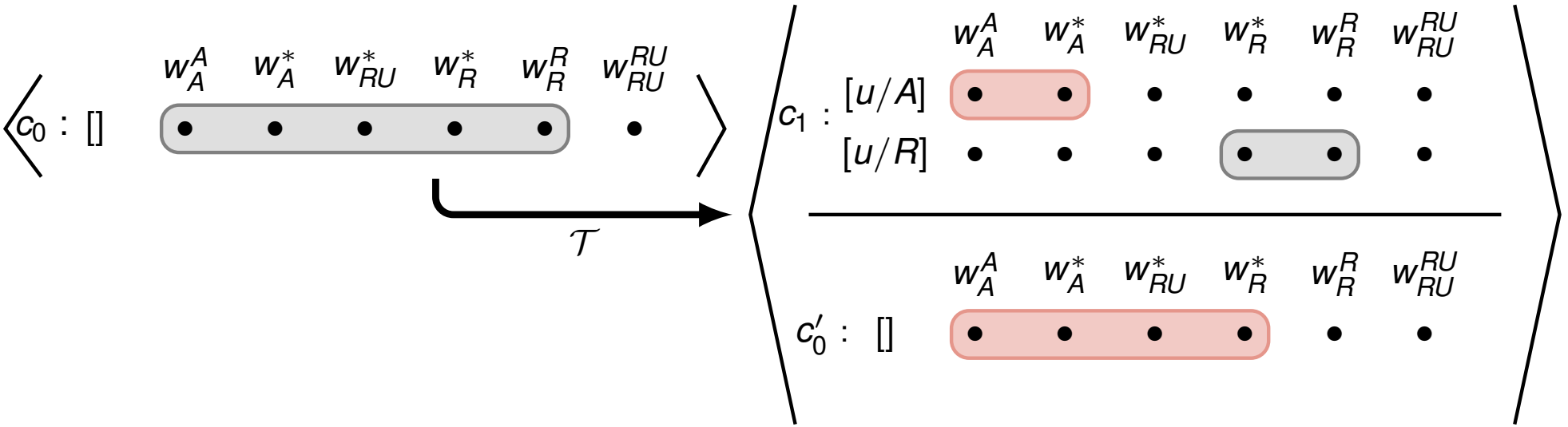

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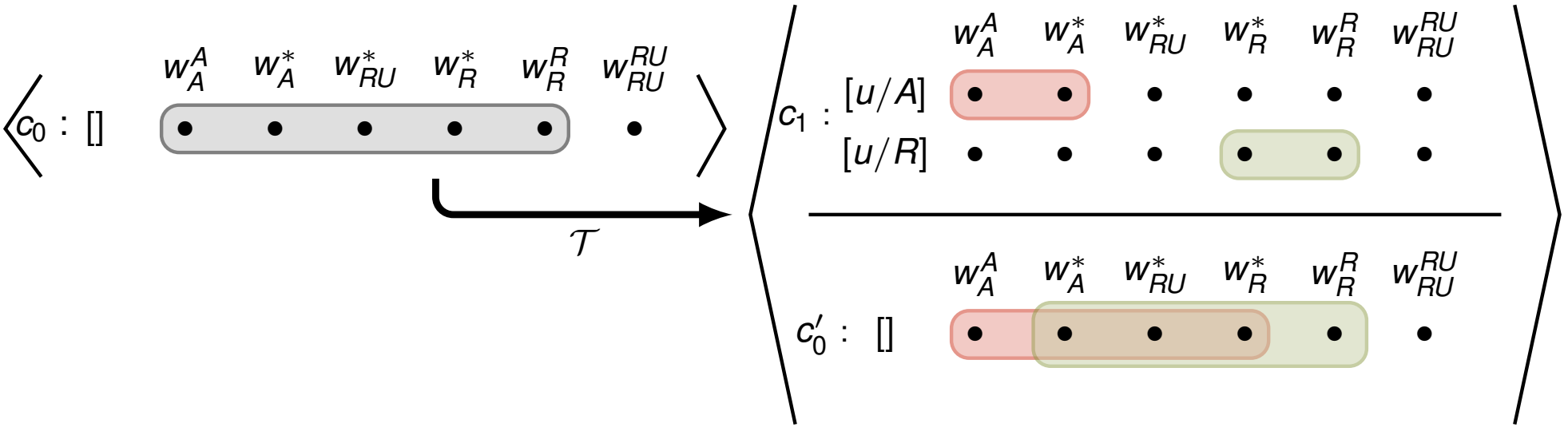

- $\dagger$: presupposition of non-informativeness
- Global presupposition obviated : c_0 can include w_{RU}^*
- Local presupposition : c_0 must exclude w_{RU}^{RU}

Conclusion

Modal Dynamic Inquisitive Semantics

- Weakening of exhaustivity
- Weakening of uniqueness presupposition
- Externally static $\Diamond$
- Capturing modal subordination
- Uniform treatment of modalized questions and conditional questions

Future prospects :

- Enable irrealis mood

Exhaustivity Weakening

(9) a. SITUATION : *Mary would like to experiment with two or three patients with eczema. She asks Ann, who knows the medical files of all patients with skin conditions in the hospital, a question.*

b. Which patients have eczema ?

c. Which patients could I invite for my experiment ?

Formulas

Pushing and percolating :

$$\begin{aligned} \text{PUSH}\,\mathcal{U} &:= \quad \lambda\langle c_0,...,c_n\rangle.\ \langle c_0,...,c_n,\mathcal{U}(c_n)\rangle \\ \text{PERC}\,\mathcal{U} &:= \quad \lambda\langle c_0,...,c_{n-2},c_{n-1},c_n\rangle.\ \langle c_0[c_{n-1}\vdash\mathcal{U}],...,c_{n-2}[c_{n-1}\vdash\mathcal{U}],\mathcal{U}(c_{n-1}),c_n\rangle \end{aligned} \tag{9}$$

KAUFMANN 2000's percolation : $c[c'\vdash\mathcal{U}] = (\Box(c'\to\mathcal{U}))(c)$:
"*Learning in c that if c′ then* $\mathcal{U}(c')$"

$$\begin{aligned} c'\to\mathcal{U} &:= \quad c\mapsto\{s\in c\mid\forall t\subseteq s.\,\forall t'\in c'.\,t\sqsubseteq t'\to t'\sqsubseteq\mathcal{U}(c')\} \\ \Box\mathcal{U} &:= \quad c\mapsto\{s\in c\mid\exists s'\in\mathcal{U}(c).\,\forall\langle w,g\rangle\in s.\,\forall w'.\,w\,R\,w'\to\langle w',g\rangle\sqsubseteq s'\} \end{aligned} \tag{10}$$

Extension and subsistence :

$$\begin{array}{rcllllcll} \langle w,g\rangle & \leqslant & \langle w',g'\rangle & \text{if} & w=w'\wedge g\subseteq g' & \quad i & \sqsubseteq & s' & \text{if} & \exists j\in s'.\,i\leqslant j \\ s & \leqslant & s' & \text{if} & \forall j\in s',\exists i\in s.\,i\leqslant j & \quad s & \sqsubseteq & s' & \text{if} & s\leqslant s'\wedge(\forall i\in s.\,i\sqsubseteq s') \\ & & & & & \quad s & \sqsubseteq & c' & \text{if} & \exists s'\in c'.\,s\sqsubseteq s' \end{array} \tag{11}$$

Presupposition operator :

$$\dagger\,\mathcal{S} := \tau\mapsto\begin{cases} \mathcal{S}(\tau) & \text{if } \forall i<|\tau|.\ \bigcup\tau_i\sqsubseteq\bigcup\mathcal{S}(\tau)_i \\ \text{undefined} & \text{otherwise} \end{cases} \tag{12}$$

Possible extension

- KRATZER 1991's modal base f and ordering source ω
- Restrictor context c_r (for irrealis mood)
- Modal structure $\eta = \langle f, \omega, c_r \rangle$
- Ideal elements at w : $\text{Ideal}_{\omega(w)}((\bigcup c_r) \cap (\bigcap f(w))) \subseteq W$

Possibility function :

$$\begin{aligned} \mathbb{P}_{\langle f,\omega,c_r\rangle}(s) := &\quad \{w \in W \mid s \cap \text{Ideal}_{\omega(w)}((\cup c_r) \cap (\bigcap f(w))) \neq \emptyset\} \\ [\![\Diamond_\eta \varphi]\!]_{\text{int}} = &\quad \mathbb{P}_\eta([\![\varphi]\!]_{\text{int}}) \end{aligned} \tag{13}$$

Extending Modal $\text{Inq}^{\text{D}}_{\text{B}}$:

$$\Diamond_\eta \mathcal{U} := c \mapsto \{s \in c \mid \exists s' \in \mathcal{U}(c_r^\eta).\ \text{WC}(s) \subseteq \mathbb{P}_\eta(\text{WC}(s'))\} \tag{14}$$